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Submergence in a Two-Foot Parshall Flume

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NOMENCLATURE

Symbol	Definition
A	Cross-sectional area of flow, ft.^{2}
F_{max}	Maximum Froude number in the flume, dimensionless
g	Acceleration due to gravity, 32.2 ft./sec.^{2}
H_a	Depth of flow in a Parshall flume located two-thirds of the length of the converging entrance section upstream from the throat crest, ft.
H_b	Depth of flow in a Parshall flume measured at a particular referenced point in the throat, ft.
H_c	Depth of flow in a Parshall flume measured at the flume bottom on the right side at a distance of 6 inches upstream from the downstream sill, ft.
H_d	Depth of flow in a Parshall flume measured at the same elevation as H_c but in the downstream wing wall 3 inches to the right of the flume wall, ft.
h_1	Depth of flow upstream from the flume, ft.
h_4	Depth of flow downstream from the flume, ft.
h_m	Minimum depth of flow in the throat, ft.
Q	Actual discharge, cfs.
V	Average velocity, fps.

PURPOSE OF STUDY

The primary objective in this study was to ascertain the validity of the method of analyzing submergence developed by Hyatt (1965) in a standard 2-foot Parshall flume. The method of analyzing submergence was first developed for a trapezoidal flume (Hyatt, 1965), was later verified for a rectangular flume (Skogerboe, Walker and Robinson, 1965), and has been shown by the authors to be valid for small Parshall flumes (Skogerboe, Hyatt, Johnson, and England, 1965). In view of previous findings, it was felt the method would also be valid for large Parshall flumes, and for this purpose the 2-foot flume was selected.

One other objective of the study was to analyze the possibility that another, or possibly better, point of downstream measurement might be found. To accomplish this, two other points, designated c and d, were selected downstream. The downstream depth measurement is usually taken in the throat at a referenced point designated b.

The resulting equations and calibration curve are listed in this report.

EXPERIMENTAL FACILITIES

A commercial fabricated steel 2-foot Parshall flume was used. The actual throat width was measured to be 1.975 feet instead of the designated 2-foot dimension. All computations were based on a throat width of 1.975 feet, but the flume elsewhere in this report is referred to as a 2-foot flume for ease of description.

The 2-foot Parshall flume was placed in the 5-foot deep by 5-foot wide flume located in the Fluid Mechanics Laboratory at Utah State University (Figures 1 and 2).

Three pumps were used which were capable of delivering a maximum flow rate of approximately eight cubic feet per second (cfs). The flow rate was regulated by varying the number of pumps on the line and by means of a valve located on the line as it entered the laboratory.

The flow passed through the flume and discharged into weighing tanks, where the water was weighed over a given time period to obtain the flow rate. The water was then discharged from the weighing tanks into the sump, where it was recirculated.

Depth measurements were made by the use of a point gage in stilling wells. The wells are designated as a, b, c, and d--traveling downstream--and are shown in Figure 1.

A tailgate was placed downstream from the Parshall flume (Figure 2) to regulate tailwater depth and thereby control and vary the degrees of submergence for each flow rate.

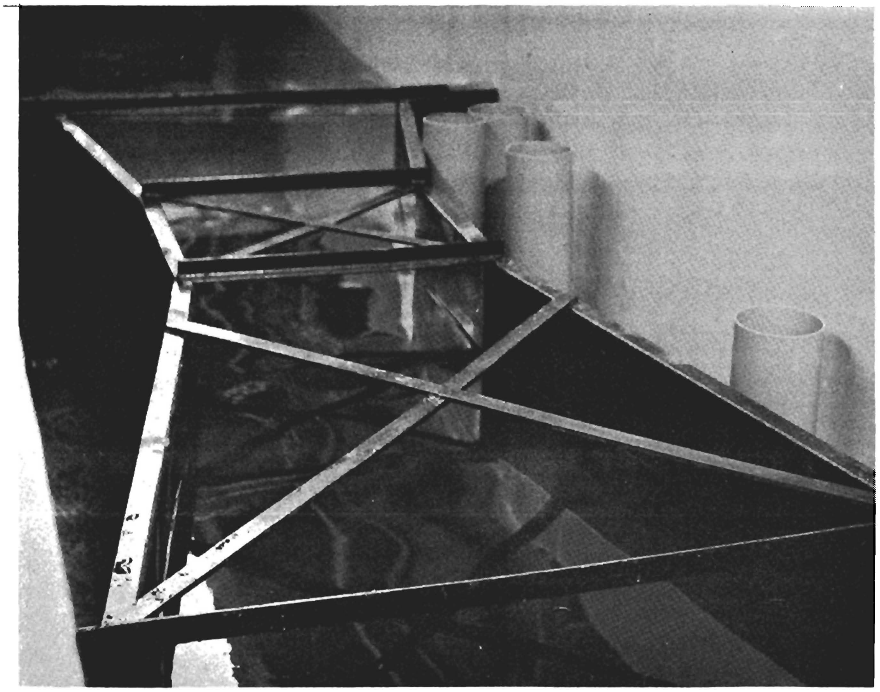

Figure 1

Two-foot fabricated steel Parshall measuring flume with wells a, b, c, and d.

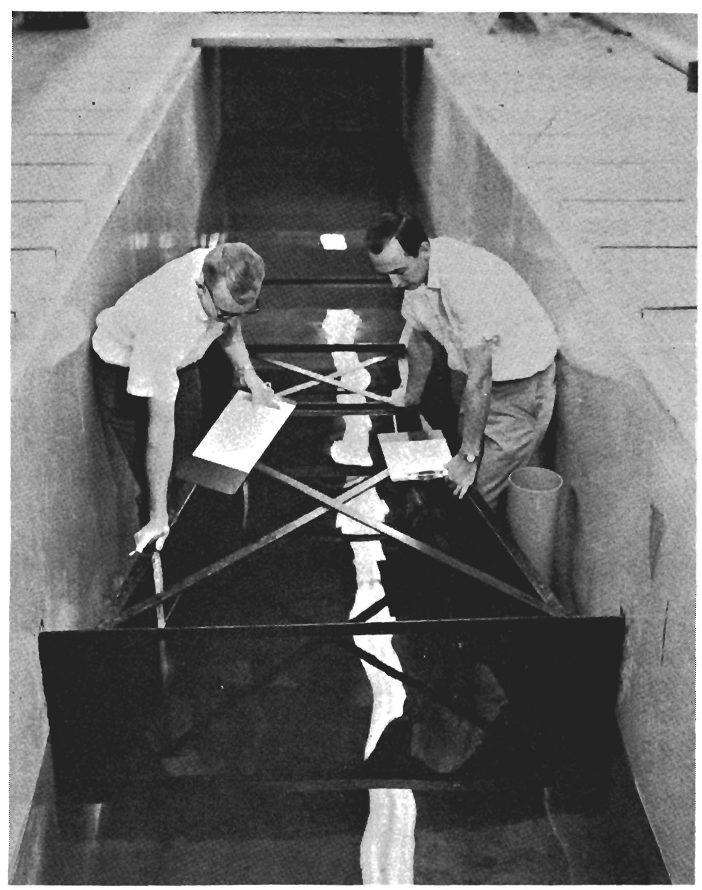

Figure 2

Submerged flow in a two-foot Parshall measuring flume.

SUBMERGED FLOW ANALYSIS

Submerged flow is said to exist in any Venturi flume if the flow in the flume fails to pass through critical depth. When submerged flow conditions exist, the stage-discharge relationship developed for free flow conditions is no longer valid and another method for analyzing the flow must be used. One method of analyzing the submerged flow problem is that developed at Utah State University. The parameters involved in utilizing this method were developed for trapezoidal measuring flumes and were later verified for rectangular measuring flumes and small Parshall flumes.

The parameters describing submerged flow are submergence, h_4/h_1, where h_4 is the flow depth downstream from the flume and h_1 the flow depth upstream from the flume; the maximum Froude number occurring in the flume, F_{max}; and a energy loss parameter relating conditions at three important cross sections as defined by $(h_1 - h_4)/h_m$, where h_m is the minimum flow depth occurring in the throat.

The parameters involved in submerged flow can be obtained from dimensional analysis as follows:

$$V = f\,(g,\ h_1,\ h_4,\ h_m) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ 1$$

With five independent quantities and two dimensions, three pi-terms are derived.

$$\pi_1 = \frac{V}{\sqrt{g\,h_m}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ 2$$

$$\pi_2 = \frac{h_4}{h_1} \quad \text{. . . . . 3}$$

$$\pi_3 = \frac{h_1 - h_4}{h_m} \quad \text{. . . . . 4}$$

Equation 2 can be modified by replacing V with Q/A so that

$$\pi_1 = \frac{Q}{A\sqrt{g\,h_m}} = F_{max} \quad \text{. . . . . 5}$$

A typical relationship between the three pi-terms F_{max}, h_4/h_1, and $(h_1-h_4)/h_m$ is illustrated in Figure 3 (Hyatt, 1965). The energy loss parameter was plotted on the log scale as the ordinate. Plotted on the abscissa are submergence on a cartesian scale and maximum Froude number on a log scale. The two resulting equations, relating these pi-terms are:

$$\frac{h_1 - h_4}{h_m} = 0.300\, F_{max}^{2.38} \quad \text{. . . . . 6}$$

and

$$\frac{h_4}{h_1} = \frac{0.99}{10^{0.34\,(h_1 - h_4)/h_m}} \quad \text{. . . . . 7}$$

When three-dimensional log-log plots relating h_m, F_{max}, and Q are prepared, it is possible to relate minimum depth and the maximum Froude number to the discharge, resulting in the equation,

$$Q = 34.7\, F_{max}\, h_m^{1.74} \quad \text{. . . . . 8}$$

By combining Equations 6, 7, and 8, the relationship describing submerged flow for a particular trapezoidal flume is obtained.

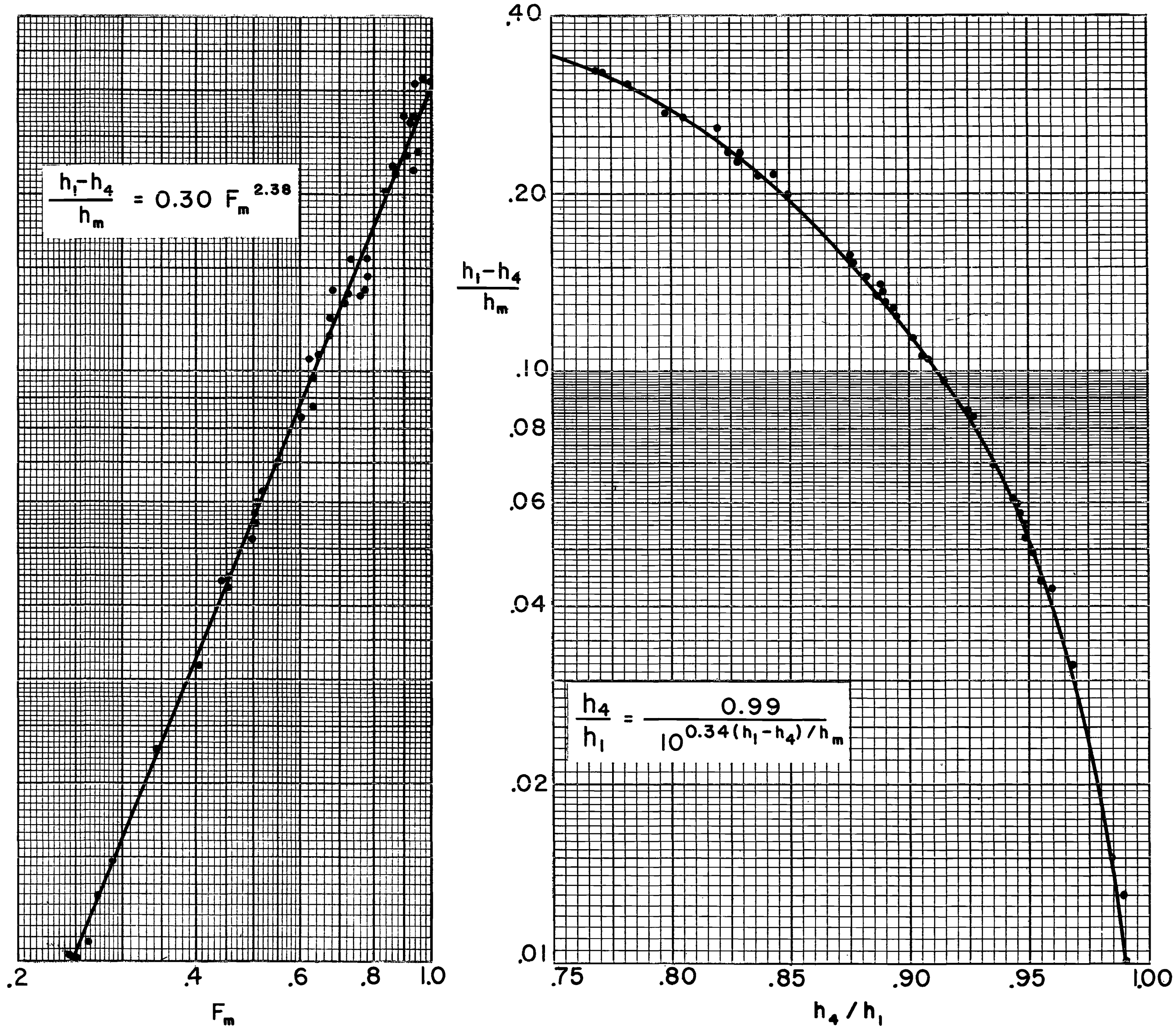

Figure 3 Relationship between pi-terms

$$Q = \frac{-13.83\ (h_1 - h_4)^{1.74}}{\left(\log \frac{h_4}{h_1} + 0.0044\right)^{1.32}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad 9$$

The equation developed by Skogerboe, Walker, and Robinson (1965) for a particular rectangular flume was

$$Q = \frac{-46.6\ (h_1 - h_4)^{1.53}}{\left(\log \frac{h_4}{h_1} + 0.0044\right)^{1.02}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad 10$$

The equations developed for small Parshall flumes are of the form

$$Q = \frac{-C\ (H_a - H_b)^{1.55}}{\log \frac{H_b}{H_a} + 0.0044} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad 11$$

where C values of 0.295, 0.614, and 0.953 were obtained for the 1-, 2-, and 3-inch Parshall flumes respectively. Although these equations (Equations 9, 10, and 11) are applicable only for the particular flumes studied, they do show that only the upstream and downstream depths need to be measured to determine the discharge under submerged flow conditions.

The approach to submergence as described above was applied to the 2-foot Parshall flume located in the laboratory. The downstream depth was measured at three locations, H_b (usual point of downstream measurement in a Parshall flume), H_c, and H_d. Point c is located at the flume bottom on the right side a distance of 6 inches upstream from the downstream sill. Point d, located at the same elevation as point c, was placed in the downstream wing wall 3 inches to the right of the flume wall.

EXPERIMENTAL FINDINGS

From the data taken in the laboratory at Utah State University, the method of analyzing submergence previously described in this report was found to be valid for the 2-foot Parshall flume not only at point b, but at points c and d as well. The discharge equation for point b is

$$Q = \frac{-5.88\ (H_a - H_b)^{1.55}}{\left(\log \frac{H_b}{H_a} + 0.0044\right)^{1.14}} \quad \dots\dots\dots\dots \quad 12$$

For point c, the equation becomes

$$Q = \frac{-5.22\ (H_a - H_c)^{1.55}}{\left(\log \frac{H_c}{H_a} + 0.0044\right)^{1.21}} \quad \dots\dots\dots\dots \quad 13$$

The equation for point d is

$$Q = \frac{-4.83\ (H_a - H_c)^{1.55}}{\left(\log \frac{H_d}{H_a} + 0.0044\right)^{1.25}} \quad \dots\dots\dots\dots \quad 14$$

Figure 4 shows the submerged calibration curve for the 2-foot Parshall flume as developed from the data taken at point b. Similar calibration curves could be developed for points c and d but were not done so in this study. The data collected at points b, c, and d proved to be of comparable accuracy. The water level in the b stilling well flucuated more than the water levels in stilling wells c and d, but the accuracy obtained from the submergence plots was essentially equal. Consequently,

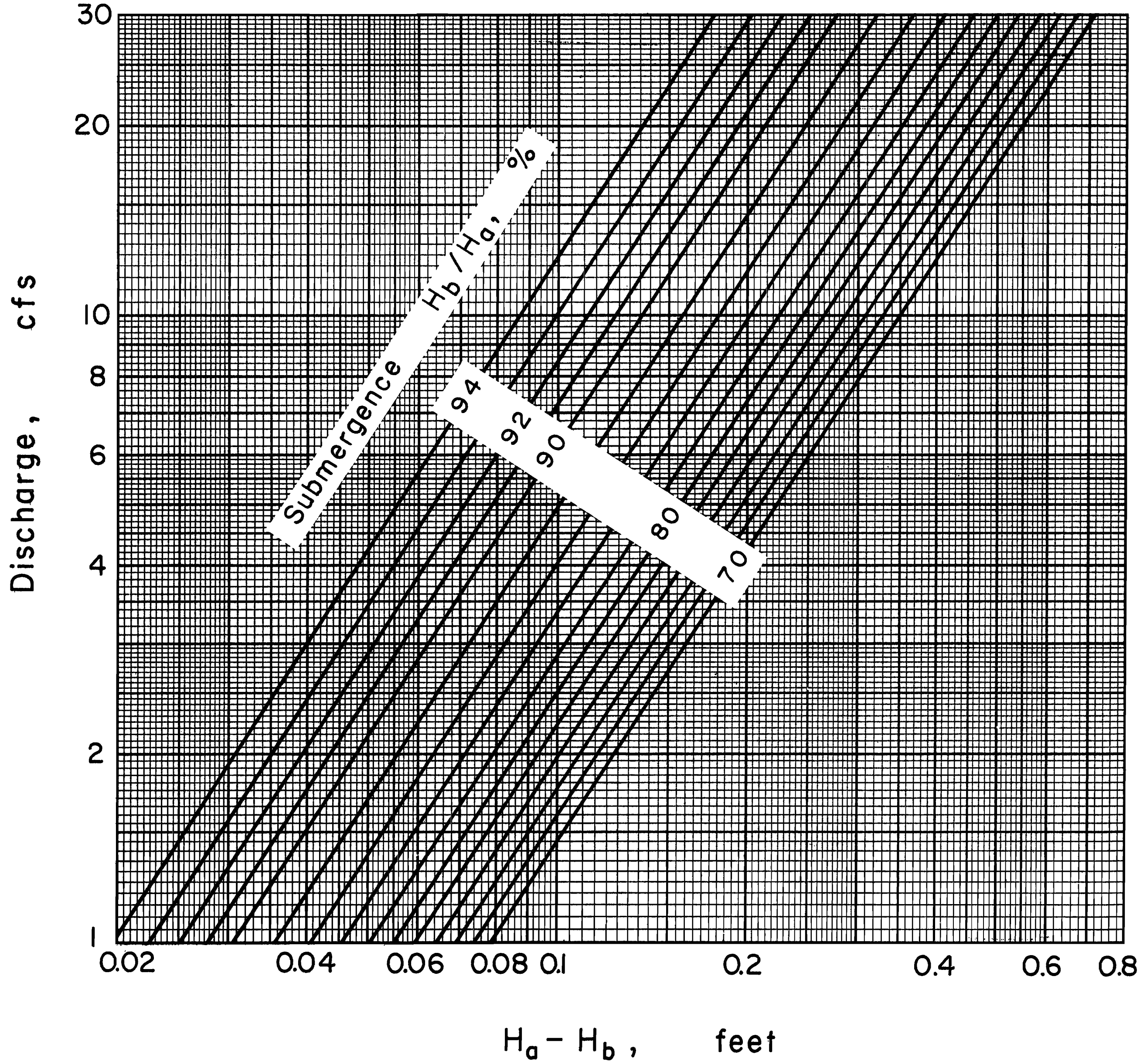

Figure 4 Submerged calibration curves for two-foot Parshall flumes.

it was felt by the authors that there was no particular advantage in changing the point of downstream depth measurement to points c or d because of the common usage of point b in evaluating submergance in Parshall flumes.

TRANSITION FROM FREE TO SUBMERGED FLOW

The form of the equations for both free and submerged flow provide a unique solution for the submergence, H_b/H_a, at which the transition from free to submerged flow occurs. To illustrate, the free and submerged flow equations for the 2-foot Parshall flume can be set equal to one another to obtain the transition point. The usual free flow equation for a 2-foot Parshall flume is $Q = 8.0\ H_a^{1.55}$, but in this study with the flume used having a throat width of 1.975 feet, the free flow equation is

$$Q = 7.90\ H_a^{1.55} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 15$$

Hence, when Equations 12 and 15 are equated

$$7.90\ H_a^{1.55} = \frac{-5.88\ (H_a - H_b)^{1.55}}{\left(\log \frac{H_b}{H_a} + 0.0044\right)^{1.14}}$$

$$\left(\log \frac{H_b}{H_a} + 0.0044\right)^{1.14} = 0.745 \left(1 - \frac{H_b}{H_a}\right)^{1.55}$$

The solution, obtained by trial and error, is

$$\frac{H_b}{H_a} = 0.66$$

Thus, free flow will exist in a 2-foot Parshall flume when the submergence is less than 66 percent and for submergences greater than 66 percent the flow becomes submerged. The solution of the free and submerged flow equations for points c and d in the 2-foot Parshall flume studied results in a critical submergence of 74.5 percent for point c and 74 percent for point d.

CONCLUSIONS

The Parshall flume is a reliable method of water measurement giving fairly accurate results. Such a flume is capable of operating under both free and submerged flow. The transition submergence (H_b /H_a) from free to submerged flow is generally accepted at about 70 percent (Parshall, 1941). Utilizing the method of submerged flow analysis developed by Hyatt (1965), which was found to be valid for the 2-foot Parshall flume, the transition point was found to be 66 percent for the 2-foot flume. Transition points were found to be 74.5 percent and 74.0 percent at points c and d, respectively. Using this method of submerged flow analysis, submerged flow calibration curves can be developed for points b, c, and d, but were developed for only point b because (1) b is the generally accepted point of downstream depth measurement, and (2) points c and d did not improve the accuracy of the submerged flow plots.

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APPENDIX

Data and Computations

Table 1. Basic measurements

Run No.	Q	H_a	H_b	H_c	H_d
1	1.215	0.410	0.388	0.392	0.395
2	1.196	0.379	0.349	0.357	0.378
3	1.196	0.342	0.299	0.311	0.308
4	1.196	0.329	0.276	0.286	0.284
5	1.196	0.319	0.253	0.263	0.263
6	1.196	0.313	0.230	0.251	0.252
7	1.196	0.311	0.266	0.245	0.245
8	1.750	0.503	0.473	0.496	0.483
9	1.750	0.429	0.385	0.398	0.393
10	1.750	0.393	0.310	0.337	0.334
11	1.750	0.382	0.291	0.311	0.310
12	1.750	0.442	0.401	0.414	0.412
13	1.750	0.381	0.290	0.308	0.305
14	1.750	0.381	0.273	0.299	0.299
15	3.920	0.861	0.813	0.841	0.840
16	3.920	0.778	0.718	0.748	0.745
17	3.920	0.743	0.670	0.708	0.712
18	3.920	0.706	0.615	0.661	0.653
19	3.920	0.686	0.579	0.628	0.619
20	3.920	0.664	0.528	0.586	0.579
21	3.920	0.652	0.543	0.569	0.565
22	3.920	0.644	0.501	0.533	0.532
23	3.920	0.642	0.484	0.522	0.518
24	3.920	0.642	0.467	0.506	0.504
25	2.970	0.721	0.686	0.702	0.699
26	2.970	0.631	0.580	0.601	0.596
27	2.970	0.553	0.443	0.486	0.479
28	2.970	0.539	0.393	0.430	0.431
29	7.400	1.203	1.131	1.178	1.173
30	6.410	1.245	1.197	1.220	1.217
31	6.410	1.121	1.032	1.086	1.080
32	6.860	1.092	0.992	1.051	1.057
33	6.860	1.065	0.961	1.027	1.024
34	6.860	1.035	0.910	0.982	0.972
35	6.860	1.003	0.853	0.934	0.925

Table 1. Continued

Run No.	Q	H_a	H_b	H_c	H_d
36	6.860	0.977	0.805	0.893	0.881
37	6.860	0.945	0.810	0.845	0.842
38	6.860	0.932	0.742	0.782	0.781
39	6.860	0.924	0.688	0.729	0.729
40	4.490	0.856	0.788	0.827	0.824
41	4.450	0.800	0.712	0.761	0.756
42	4.290	0.763	0.657	0.707	0.697
43	4.430	0.724	0.580	0.639	0.636
44	4.390	0.700	0.552	0.588	0.588
45	4.390	0.700	0.505	0.546	0.547
46	5.650	0.823	0.582	0.620	0.627
47	5.650	0.840	0.701	0.739	0.724
48	5.650	0.828	0.621	0.662	0.662
49	2.700	0.720	0.690	0.704	0.704
50	2.700	0.683	0.638	0.666	0.664
51	2.700	0.665	0.628	0.642	0.641
52	2.700	0.616	0.572	0.587	0.586
53	2.700	0.573	0.512	0.539	0.534
54	2.700	0.551	0.472	0.505	0.501
55	2.700	0.533	0.434	0.475	0.479
56	2.700	0.530	0.473	0.463	0.457
57	2.700	0.522	0.411	0.451	0.449
58	2.700	0.514	0.396	0.428	0.426
59	2.700	0.512	0.367	0.411	0.407
60	5.300	1.118	1.066	1.093	1.095
61	5.300	1.027	0.956	0.997	0.992
62	5.300	0.914	0.807	0.871	0.870
63	5.300	0.869	0.737	0.810	0.797
64	5.300	0.827	0.662	0.737	0.726
65	1.330	0.457	0.438	0.443	0.445
66	1.330	0.359	0.314	0.326	0.323
67	1.330	0.383	0.350	0.360	0.358
68	1.330	0.371	0.334	0.344	0.342
69	1.330	0.352	0.301	0.314	0.312
70	1.350	0.346	0.284	0.298	0.296

Table 1. Continued

Run No.	Q	H_a	H_b	H_c	H_d
71	1.350	0.341	0.274	0.293	0.290
72	1.350	0.340	0.269	0.287	0.284
73	1.350	0.338	0.260	0.280	0.278
74	1.350	0.337	0.249	0.275	0.273
75	1.350	0.335	0.242	0.268	0.266
76	1.340	0.332	0.235	0.255	0.256

Table 2. Computation of parameters

Run No.	Q	H_b/H_a	H_c/H_a	H_d/H_a	H_a-H_b	H_a-H_c	H_a-H_d
1	1.215	0.946	0.956	0.963	0.022	0.018	0.015
2	1.196	0.920	0.942	0.997	0.030	0.022	0.001
3	1.196	0.874	0.909	0.901	0.043	0.031	0.034
4	1.196	0.838	0.869	0.863	0.053	0.043	0.045
5	1.196	0.793	0.824	0.824	0.066	0.056	0.056
6	1.196	0.735	0.802	0.805	0.083	0.062	0.061
7	1.196	0.727	0.788	0.788	0.085	0.066	0.066
8	1.756	0.940	0.986	0.960	0.030	0.007	0.020
9	1.756	0.907	0.937	0.932	0.041	0.028	0.030
10	1.756	0.897	0.928	0.916	0.044	0.031	0.036
11	1.756	0.789	0.857	0.850	0.083	0.056	0.059
12	1.756	0.762	0.814	0.812	0.091	0.071	0.072
13	1.756	0.761	0.808	0.801	0.091	0.073	0.076
14	1.756	0.717	0.785	0.785	0.108	0.082	0.082
15	3.945	0.944	0.977	0.976	0.048	0.020	0.021
16	3.945	0.923	0.961	0.958	0.060	0.030	0.033
17	3.945	0.902	0.953	0.958	0.073	0.035	0.031
18	3.945	0.871	0.936	0.925	0.091	0.045	0.053
19	3.945	0.844	0.915	0.902	0.107	0.058	0.067
20	3.945	0.795	0.883	0.872	0.136	0.078	0.085
21	3.945	0.833	0.873	0.866	0.109	0.083	0.087
22	3.929	0.778	0.828	0.826	0.143	0.111	0.112
23	3.929	0.754	0.813	0.807	0.158	0.120	0.124
24	3.929	0.727	0.788	0.785	0.175	0.136	0.138
25	2.968	0.951	0.974	0.969	0.037	0.019	0.022
26	2.968	0.919	0.952	0.944	0.050	0.030	0.035
27	2.968	0.801	0.879	0.866	0.110	0.067	0.054
28	2.968	0.729	0.798	0.800	0.146	0.109	0.108
29	7.396	0.940	0.979	0.975	0.072	0.025	0.030
30	6.410	0.961	0.980	0.978	0.043	0.025	0.028
31	6.410	0.921	0.969	0.963	0.089	0.035	0.041
32	6.868	0.908	0.962	0.968	0.100	0.041	0.035
33	6.868	0.902	0.964	0.962	0.104	0.038	0.041
34	6.868	0.879	0.949	0.939	0.125	0.053	0.063
35	6.868	0.850	0.931	0.922	0.150	0.069	0.028

Table 2. Continued

Run No.	Q	H_b/H_a	H_c/H_a	H_d/H_a	H_a-H_b	H_a-H_c	H_a-H_d
36	6.868	0.824	0.914	0.902	0.172	0.084	0.096
37	6.868	0.857	0.894	0.891	0.135	0.100	0.103
38	6.868	0.796	0.839	0.838	0.190	0.150	0.151
39	6.868	0.744	0.789	0.789	0.236	0.195	0.195
40	4.493	0.920	0.966	0.963	0.068	0.029	0.032
41	4.452	0.890	0.951	0.945	0.088	0.039	0.044
42	4.293	0.861	0.927	0.913	0.106	0.056	0.066
43	4.431	0.801	0.883	0.878	0.144	0.085	0.088
44	4.390	0.789	0.840	0.840	0.148	0.112	0.112
45	4.390	0.721	0.780	0.781	0.195	0.154	0.153
46	5.656	0.707	0.753	0.762	0.241	0.203	0.196
47	5.656	0.834	0.880	0.862	0.139	0.101	0.116
48	5.656	0.750	0.800	0.800	0.207	0.166	0.166
49	2.710	0.958	0.978	0.978	0.030	0.016	0.016
50	2.710	0.939	0.975	0.973	0.045	0.017	0.019
51	2.710	0.947	0.965	0.964	0.037	0.023	0.024
52	2.710	0.929	0.954	0.952	0.044	0.029	0.030
53	2.710	0.893	0.941	0.932	0.061	0.034	0.039
54	2.710	0.857	0.916	0.909	0.081	0.048	0.052
55	2.710	0.815	0.893	0.898	0.099	0.058	0.054
56	2.710	0.798	0.874	0.881	0.107	0.067	0.073
57	2.710	0.788	0.865	0.862	0.111	0.071	0.073
58	2.710	0.770	0.833	0.829	0.118	0.086	0.088
59	2.710	0.710	0.803	0.795	0.145	0.101	0.105
60	5.300	0.954	0.978	0.979	0.052	0.025	0.023
61	5.300	0.933	0.971	0.967	0.071	0.030	0.035
62	5.300	0.883	0.955	0.953	0.107	0.043	0.044
63	5.300	0.848	0.932	0.916	0.132	0.059	0.070
64	5.300	0.800	0.891	0.878	0.165	0.090	0.101
65	1.330	0.958	0.969	0.974	0.019	0.014	0.012
66	1.330	0.875	0.908	0.900	0.045	0.033	0.036
67	1.330	0.911	0.938	0.933	0.033	0.023	0.025
68	1.330	0.900	0.927	0.922	0.037	0.027	0.029
69	1.330	0.855	0.893	0.887	0.051	0.038	0.040
70	1.350	0.821	0.862	0.856	0.062	0.048	0.050

Table 2. Continued

Run No.	Q	H_b/H_a	H_c/H_a	H_d/H_a	H_a-H_b	H_a-H_c	H_a-H_d
71	1.350	0.802	0.859	0.851	0.068	0.048	0.051
72	1.350	0.792	0.844	0.835	0.071	0.053	0.056
73	1.350	0.767	0.827	0.821	0.078	0.058	0.060
74	1.350	0.739	0.816	0.811	0.088	0.062	0.064
75	1.350	0.722	0.800	0.794	0.093	0.067	0.069
76	1.340	0.707	0.769	0.772	0.097	0.077	0.076